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## LETTER TO THE EDITOR

# Violation of Bell's inequalities and distillability for $N$ qubits

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## Abstract

We consider quantum systems composed of  $N$  qubits, and the family of all Bell's correlation inequalities for two two-valued measurements per site. We show that if an  $N$ -qubit state  $\rho$  violates any of these inequalities, then it is at least bipartite distillable. Indeed there exists a link between the amount of Bell's inequality violation and the degree of distillability. Thus, we strengthen the interpretation of Bell's inequalities as detectors of useful entanglement.

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## 1. Introduction

Quantum mechanics predicts remarkable correlations between the outcomes of measurements on sub-systems (particles) of a composed system. This prediction is a consequence of the interplay between the superposition principle and the tensor product structure of quantum mechanics, that allows states to be built that cannot be written as probabilistic mixtures of products of states of each sub-system. Such states are called non-separable or *entangled*. This letter concerns the characterization of entanglement for multi-partite systems. This is a complex problem. The variety of known partial results suggests that there may not be a unique characterization of entanglement. We show a close connection between two features of entangled states, *distillability* and *Bell's inequality violation* in systems of  $N$  quantum bits (qubits). But first, let us begin by reviewing the notions of separability, distillability and quantum non-locality.

*Separability.* A pure state  $|\Psi\rangle$ , shared by  $N$  partners, is separable when it can be written as the tensor product of pure states of each subsystem, i.e.  $|\Psi\rangle = |\psi_1\rangle \otimes \cdots \otimes |\psi_N\rangle$ . Thus, for these states there are no correlations between the different sub-systems. For the mixed-state case, a density matrix,  $\rho$ , is separable when it can be expressed as the convex combination of separable pure states,  $\rho = \sum_i p_i |\Psi_i^s\rangle\langle\Psi_i^s|$ , where  $|\Psi_i^s\rangle$  are product states  $\forall i$ . States that are

not separable are called entangled. It is clear from these definitions that if the initial state of the system is separable and the partners are allowed to use only local operations and classical communication (LOCC), then definitely no entangled state can be prepared.

*Distillability* [1]. An entangled state  $\rho$  is distillable if some partners sharing arbitrarily many copies of the state can produce a few maximally entangled states<sup>3</sup> using only LOCC. Once two partners share a maximally entangled state, they can run quantum communication protocols, such as teleportation [2] or quantum key distribution [3]. Thus distillability is a measure of the ‘usefulness’ of a state: if a state is distillable, we can extract from its many copies the kind of entanglement needed to implement quantum communication protocols.

A necessary (and conjectured not sufficient [5, 6]) condition for distillability in the bipartite case is the non-positivity of the partial transpose (NPPT): the partial transpose of  $\rho^4$  must have at least one negative eigenvalue [4].

*Quantum non-locality.* If we focus on their preparation, all entangled states have some form of quantum non-locality, in the sense that they cannot be prepared without using either non-local quantum operations or quantum channels. However in this letter we use the term quantum non-locality for denoting the impossibility of reproducing the correlations observed in some quantum states of a multi-partite system by means of local variable theories (LV).

Assume that a quantum state  $\rho$  is prepared at a given location and distributed among  $N$  partners at different locations. The partners can apply a LOCC protocol that transforms  $M$  copies of it into a new state,  $\rho'$ , with some probability. Then they perform a sequence of measurements on it. We will call a state *local* if the statistics of the outcomes can be reproduced by an LV model for all these measurement and LOCC protocols. If there exists a protocol such that the outcomes cannot be reproduced by any LV model, we say that the state exhibits some *quantum non-locality*. The last step for the detection of quantum non-locality consists of checking if the measurement outcomes violate some constraints, known as Bell’s inequalities [7, 8], that any local variable model satisfies. If the state  $\rho'$  violates a Bell’s inequality it is clearly quantum non-local. We will also say  $\rho$  is quantum non-local since it can be transformed into  $\rho'$  without non-local quantum resources.

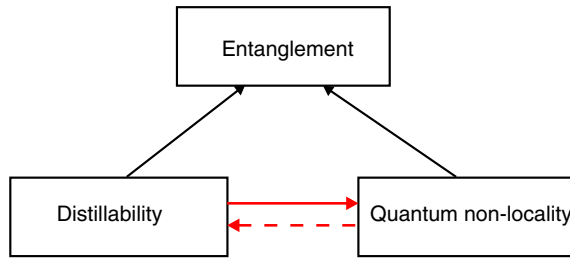
No necessary and sufficient conditions for quantum non-locality are known; in particular, nobody knows whether all entangled states are non-local. Since the work of Werner [9] (and its extension by Barrett [10]), it has been known that there exist entangled states that do not violate any Bell’s inequality. This does not mean that they cannot violate a Bell’s inequality after LOCC filtering protocols [11]. However, it is conjectured that some of the states discussed in [9, 10] are not distillable. Hence, they may not exhibit any quantum non-locality although they are entangled.

We have completed our review of distillability and quantum non-locality. Let us consider the first link between these two notions. From the previous definitions it follows that if  $\rho$  is distillable then it is also quantum non-local. In fact, if  $\rho$  is distillable, then the distilled maximally entangled states violate a Bell’s inequality for suitable measurements and in this way reveal the quantum non-locality of the initial state.

The simplest composite scenario corresponds to a two-qubit system. There Bell’s inequality due to Clauser *et al* [12] plays an essential role. In the case of two two-outcome measurements per site, it provides a necessary and sufficient condition for the existence of

<sup>3</sup> A pure state  $|\Psi\rangle \in \mathbb{C}^d \otimes \mathbb{C}^d$  is a maximally entangled state when it can be written as  $|\Psi\rangle = 1/\sqrt{d} \sum_{i=1}^d |ii\rangle$  where  $\{|i\rangle\}$  are orthonormal local bases. For distillability it is sufficient to demand that a two-qubit maximally entangled state, also called a singlet, can be obtained by LOCC.

<sup>4</sup> Consider an operator,  $O$ , that acts on  $\mathbb{C}^{d_1} \otimes \mathbb{C}^{d_2}$ . The partial transposition of  $O$  with respect to the first subsystem in the basis  $\{|1\rangle, \dots, |d_1\rangle\}$  is given by  $O^{T_1} \equiv \sum_{i,j=1}^{d_1} \langle i|O|j\rangle |j\rangle\langle i|$ .



**Figure 1.** The figure summarizes the known implications between entanglement, distillability and quantum non-locality. If a state is distillable or it cannot be described by an LV model, then it is entangled. Bound entangled states show that not all entangled states are distillable. Whether all entangled states have quantum non-locality is an open question. According to our definitions, distillability implies quantum non-locality. In this letter we explore whether the converse is also true (dashed line) in systems of  $N$  qubits.

a local variable model [13]. Moreover, all entangled states of two qubits can be distilled to the singlet form [14] (figure 1). In summary, the physics of entanglement for two qubits conveys the intuitive idea that all entangled states shall ultimately show some form of quantum non-locality and can be used as resources in quantum information protocols. However, as soon as one moves to more complex quantum systems, these results are not yet proved or are proved to be wrong! The most surprising feature is the existence of *bound entangled states*, which are states that are entangled but cannot be distilled [4]. Such states<sup>5</sup> seem to be ‘useless’ for quantum information protocols: the entanglement that has been used to prepare them [15] cannot be recovered (at least, without the help of additional quantum resources [16]).

In this letter, we study the link between Bell’s inequalities and distillability on a quantum system composed of  $N$  qubits. Recently, a large family of Bell’s inequalities for multipartite systems has been completely characterized; thus we can study the link with distillability on a by now well-known mathematical structure. We will in particular discuss the situation where the  $N$  qubits are shared by less than  $N$  distant parties (section 2). In section 3 we present the main results which can be summarized as follows: (i) if a state of  $N$  qubits violates any of the inequalities in the family that we consider, then one can always find at least two subgroups that can distil a singlet if quantum communication is allowed within each group; (ii) the amount of violation of the inequality provides an upper bound to the size of the subgroups; the higher the violation the smaller the subgroups; (iii) in particular, if the violation of the inequality is sufficiently large, *each pair* of qubits can distil a singlet, i.e. there is no need to establish a quantum channel. Using teleportation, this implies that any quantum state of  $N$  qubits can be distilled (‘full distillability’) by means of operations which are local with respect to all  $N$  tensor factors.

## 2. $N$ qubits as a multi-composed system

### 2.1. Multi-qubit entanglement and distillability

The classification of entanglement for multi-composed systems (here we stick to the case of  $N$  qubits) is not an easy task. Indeed, it is not even known what the fundamental types of pure-state entanglement are [17]. In a qualitative way, some extreme cases are easily

<sup>5</sup> To be precise, bound entangled states also do not exist for  $\mathbb{C}^2 \otimes \mathbb{C}^3$ .

found: on one side, full product states,  $|\psi_1\rangle \otimes \cdots \otimes |\psi_N\rangle$ ; on the other side, fully entangled states, i.e. pure states that are bipartite entangled with respect to any splitting of the parties into two groups. A known example of the latter kind of states is the  $N$ -qubit GHZ state  $|\text{GHZ}\rangle_N = 1/\sqrt{2}(|0\rangle^{\otimes N} + |1\rangle^{\otimes N})$ . But in between these extreme cases the zoology is extremely rich, and has been the object of several studies. One can tentatively define a *degree of entanglement*. A state such as  $|\text{GHZ}\rangle_n \otimes |0\rangle^{N-n}$  is clearly  $n$ -partite entangled. Extending this definition to density matrices, one can say that  $\rho$  has  $n$ -partite entanglement if, in any possible decomposition of  $\rho$  into mixtures of pure states, there is at least one state that is  $n$ -partite entangled ( $n$ -partite entanglement was needed in the state preparation).

Now, what does it mean for a multi-partite state to be distillable? As we wrote in the introduction, in the bipartite case, a state  $\rho$  is distillable when out of its possibly many copies the two parties can extract a two-qubit maximally entangled state by LOCC. It is not completely evident how to extend this definition to a multi-partite scenario. We proceed in two steps.

Let us first define *full distillability*: a quantum state shared by  $N$  parties is  $N$ -partite ('fully') distillable when it is possible to distil singlet states between any pairs of parties using LOCC. This is equivalent to the demand that any truly  $N$ -partite entangled state, in particular an  $N$ -qubit GHZ state, can be obtained by LOCC. Indeed, once all the parties are connected by singlets, one of them can locally prepare any of these states and send it to the rest by teleportation. On the other hand, if the parties share an  $N$ -partite pure entangled state, there exist local projections such that  $N - 2$  qubits project the remaining two parties into a bipartite entangled pure state [18], which is always distillable.

Consider now a state  $\rho$  of  $N$  qubits which is *not*  $N$ -partite distillable. Still it may happen that, if some of the qubits join into several groups (or establish quantum channels between them), then the state, now shared by  $L < N$  parties, is  $L$ -partite distillable [19]. That is: without using any global quantum operation between them, but just using LOCC, each pair among the  $L$  parties can distil a singlet.

Thus, it is possible to estimate the *degree of distillability* in an  $N$ -qubit state by means of the minimal size of the groups the parties have to create in order to distil singlets, i.e. pure-state entanglement, between them. For instance, in the situation shown in figure 3 (see below) this minimal size is equal to three. We have two extreme cases in this classification: (i) the parties can perform all the operation without joining, and then the state is fully distillable as was defined above, or (ii) they have to join into two groups, and then the state is said to be bipartite distillable [19]. All the other cases lie between these two possibilities.

## 2.2. The family of Bell's inequalities

A complete set of Bell correlation inequalities for  $N$ -partite systems was found by Werner and Wolf [20], and independently by Zukowski and Brukner in [21]. Every local observer,  $i = 1, \dots, N$ , can measure two dichotomic observables,  $O_i^1$  and  $O_i^2$ , whose outcomes are labelled  $\pm 1$ . Thus, after many rounds of measurements, all the parties collect a list of experimental numbers, and they can construct the corresponding list of correlated expectation values,  $E(j_1, j_2, \dots, j_N) = \langle O_1^{j_1} \otimes O_2^{j_2} \otimes \cdots \otimes O_N^{j_N} \rangle$ , where  $j_i = 1, 2$ . The general expression for the 'WWZB-inequalities' is given by a linear combination of the correlation expectation values,

$$I_N(\vec{c}) = \sum_{j_1, \dots, j_N} c(j_1, \dots, j_N) E(j_1, \dots, j_N) \leq 1 \quad (1)$$

where the conditions for the coefficients  $\vec{c}$  can be found in [20, 21]. Using another standard terminology,  $I_N(\vec{c})$  is the expectation value of the *Bell operator* [22] defined as

$$B_N = \sum_{j_1, \dots, j_N} c(j_1, \dots, j_N) O_1^{j_1} \otimes \dots \otimes O_N^{j_N} \quad (2)$$

the Bell inequality reads  $\text{Tr}(\rho B_N) \leq 1$ .

The WWZB set of inequalities (1) is complete in the following sense. If any of these inequalities is violated, the observed correlations  $E(j_1, \dots, j_N)$  do not admit a LV description. If none of these inequalities is violated, there exists a LV model for the list of data  $E(j_1, \dots, j_N)$ —but this LV model may not reproduce the correlations of another subset of observables [20].

An important member of this family of inequalities is the Mermin–Belinskii–Klyshko (MBK) inequality [23, 24]. Given that each local observer  $i = 1, \dots, N$  measures either  $O_i^1 = \hat{n}_i \cdot \vec{\sigma} \equiv \sigma(\hat{n}_i)$  or  $O_i^2 = \sigma(\hat{n}'_i)$ , the  $N$ -qubit Bell operator for these inequalities is defined recursively as

$$M_N = \frac{1}{2}[(\sigma(\hat{n}_N) + \sigma(\hat{n}'_N)) \otimes M_{N-1} + (\sigma(\hat{n}_N) - \sigma(\hat{n}'_N)) \otimes M'_{N-1}] \quad (3)$$

where  $M'_n$  is obtained from  $M_n$  interchanging  $\hat{n}_i$  and  $\hat{n}'_i$ , and  $M_1 = \sigma(\hat{n}_1)$ .

Each inequality in the WWZB family is maximally violated by the  $N$ -qubit GHZ state [20, 26]. The maximal quantum violation of the set of inequalities is obtained for the MBK one [20], with the choice of measurements optimized for the GHZ state, and it is equal to  $2^{(N-1)/2}$ . Then, quantum mechanical violations of WWZB inequalities range from one up to this value, but not all the inequalities can be violated up to this bound.

It has also been noticed [24, 25] that in the interval  $]1, 2^{(N-1)/2}]$  and for the MBK case one can define *degrees of violation* that are associated with the degrees of entanglement. Specifically: an  $N$ -qubit state in which at most  $M \leq N$  qubits are entangled cannot violate the MBK inequality by more than  $2^{(M-1)/2}$ . In other words, if the violation exceeds  $2^{(M-1)/2}$ , one can be sure that at least  $M + 1$  qubits are entangled. In particular, all the  $N$  qubits must be entangled in order to observe a violation in the highest range  $]2^{(N-2)/2}, 2^{(N-1)/2}]$ . Our main result consists on linking the amount of violation of these inequalities with the degree of distillability, which is of course a much stronger result than the link with the degree of entanglement, for all the WWZB inequalities.

### 3. Amount of violation and degree of distillability

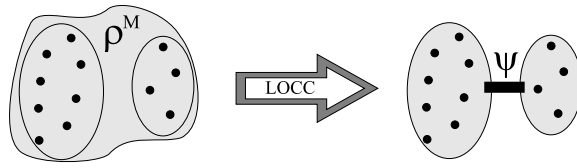
#### 3.1. The theorems

Here we give the results of our investigation. We give them in the form of two theorems and one corollary. The mathematical proofs are just sketched and are discussed elsewhere [27] in all detail.

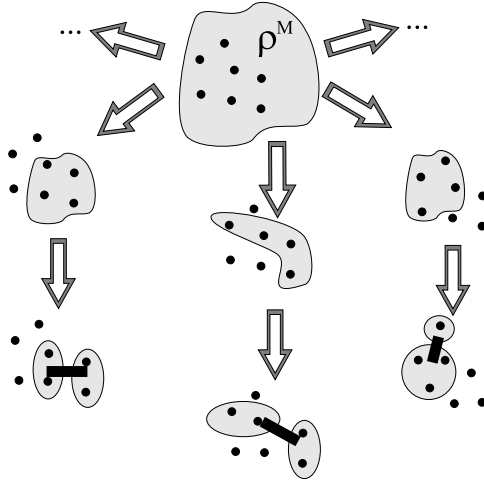
**Theorem 1.** *Any  $N$ -qubit state  $\rho$  that violates one of the WWZB inequalities is at least bipartite distillable (figure 2).*

In clear: if one can find a Bell operator  $B_N$  in the WWZB family such that  $\text{Tr}(\rho B_N) > 1$ , then there exists at least one partition of the  $N$  parties into two groups such that the two groups can distil a singlet. Loosely speaking, theorem 1 ensures that in any state that violates a WWZB Bell's inequality there is some distillable entanglement, although in order to extract it some parties may need to join.

The proof given in [27] is constructive, i.e. it provides the distillation protocol. It is a combination of three known rather technical results: (i) if a state  $\rho$  violates a WWZB



**Figure 2.** Theorem 1 illustrated for  $N = 12$  qubits. The state  $|\Psi\rangle$  is a two-qubit maximally entangled state.



**Figure 3.** Theorem 2 illustrated for  $N = 7$  qubits and a violation measured by  $p = 4$ . As in figure 2, the hollow arrows represent LOCC operations, and the thick links represent the singlet state. Any three  $N - p = 3$  qubits can perform a suitable measurement and communicate the result to the others. The four remaining qubits end up with a state which is bipartite distillable. Note that only three out of the  $C_3^7 = 7!/(3!4!) = 35$  possibilities are shown.

inequality, then there exists at least a bipartite partition such that the partial transpose of  $\rho$  with respect to this partition is negative [20]; (ii) the eigenvectors of any WWZB Bell operator are GHZ-like states [26]; (iii) there exists a protocol that allows to depolarize any  $N$ -qubit state  $\rho$  onto a state  $\rho'$  that is diagonal in a basis of GHZ-like states [19].

**Theorem 2.** Suppose that the  $N$ -qubit state  $\rho$  violates one of the WWZB inequalities by an amount of

$$\text{Tr}(\rho B_N) > 2^{\frac{N-p}{2}} \quad (4)$$

for a given integer  $p$  such that  $2 \leq p \leq N$ . Then any ensemble of  $p$  qubits can be divided into two subgroups, and a singlet can be distilled between these subgroups by means of operations which are local with respect to the  $N - p + 2$  parties (figure 3).

The proof is also constructive. If  $p = N$ , we have theorem 1: there is just one ensemble of  $p = N$  qubits, which is the full set, and we know that if the inequality is violated then the state is bipartite distillable. Suppose now that  $p \leq N - 1$ . Then one can rather easily show the following: if any of the observers measures his qubit in a suitable way and communicates

the result to the others, then the resulting conditional state  $\rho_{N-1}$  of  $N - 1$  qubits is such that (see [27] for details)

$$\mathrm{Tr}(\rho_{N-1} \tilde{B}_{N-1}) \geq \frac{1}{\sqrt{2}} \mathrm{Tr}(\rho B_N). \quad (5)$$

Here,  $\tilde{B}_{N-1}$  is a WWZB Bell operator on  $N - 1$  qubits, linked in a constructive way to the original  $B_N$ . By recursive application of this argument: we start from (4); if any  $N - p$  parties perform the suitable local measurement and communicate the result to the others, then the remaining  $p$  parties share a conditional state  $\rho_p$  that still violates a WWZB inequality. To these  $p$  qubits we apply theorem 1.

Note that in its statement theorem 1 is a corollary of theorem 2; however, the proof of theorem 2 that we have just described relies on the validity of theorem 1, which is therefore logically independent in our construction. Let us conclude with an important corollary of theorem 2:

**Corollary.** *If  $N$ -qubit state  $\rho$  violates one of the WWZB inequalities in the range*

$$2^{\frac{N-2}{2}} < \mathrm{Tr}(\rho B_N) \leq 2^{\frac{N-1}{2}} \quad (6)$$

*then  $\rho$  is fully distillable.*

This corollary can be derived independently by showing that for (6) to hold,  $\rho$  must have (up to local unitary transformations) a large overlap with the  $N$ -qubit GHZ state. In fact, the bounds given in [30] imply in particular  $\langle \mathrm{GHZ} | \rho | \mathrm{GHZ} \rangle > 2/3$  for all  $N > 3$ . When one refers to the work of Dür and Cirac [22], it is easy to see that this condition implies full distillability (in the notation of these authors,  $\langle \mathrm{GHZ} | \rho | \mathrm{GHZ} \rangle = \lambda_0^+$ ).

### 3.2. Two comments

Let us add two comments on the above results. First, we stress that the bounds given in these theorems cannot be improved on the whole WWZB class. Suppose in fact that we just know that for a state  $\rho$  there exists a WWZB Bell operator for which  $\mathrm{Tr}(\rho B_N) = 2^{(N-2)/2}$ . This state might be  $|\mathrm{GHZ}\rangle_{N-1} \otimes |0\rangle$ , which is known to reach this bound for the MBK inequality, and which is clearly not fully distillable. One might say that the criterion of theorem 2 is very simple, since it involves only one number (the amount of violation), and its roughness is the price to pay for its simplicity. Certainly, more sharp criteria could be derived if one has some knowledge of the inequality; and we have shown elsewhere [27] that if one restricts to some specific inequalities, then a refinement is indeed possible<sup>6</sup>.

The second comment concerns bound entanglement. Dür [28] was the first to note the existence of states that violate a WWZB (actually, a MBK) inequality, while being bound entangled for some partitions (in his case the most ‘natural’ partition into  $N$  qubits). This result fits well in our general scheme, on which it casts some new light. In fact, in order to have a violation, we have shown that it is necessary that *some* partitions are associated with distillable entanglement (see also [29]), while Dür’s example shows explicitly that it is not necessarily the case for all partitions. We would like to stress in this context a matter of terminology: in multi-partite systems, the notion of ‘bound entangled states that violate an inequality’ is equivocal, precisely because the entanglement may be bound for some partitions and distillable for other ones. Based on the results and the ideas described here, we strongly believe that *distillable* entanglement is responsible for the violation of a Bell’s inequality in  $N$ -qubit systems.

<sup>6</sup> Indeed we have examples where an infinitesimal Bell’s inequality violation is sufficient for fully distillability.



#### 4. Conclusion and open questions

In this work, we have discussed the complete solution of the link between distillability and the WWZB family of Bell's inequalities for  $N$  qubits. This study illustrates the complexity and the richness of the structure of multi-partite entanglement.

Of course, this work is susceptible of some extension even as far as the case of  $N$  qubits is concerned, since after all we have proved the link with distillability 'only' for the WWZB inequalities. A first direction would be to study similar connections for inequalities with more settings per site [30]. Nevertheless, note that one cannot exclude the possibility that from a quantum state violating a Bell's inequality that does not belong to this family (with more setting or outcomes) a new state can always be obtained by LOCC violating an inequality in the WWZB family. Very little is known about the structure of Bell's inequality quantum violations under LOCC transformations.

A hotter related open problem is the extension to higher dimensional systems. Even in the case of bipartite systems  $\mathbb{C}^d \otimes \mathbb{C}^d$ , where there is bipartite bound entanglement, nothing is known about the link between violation of Bell's inequalities and distillability<sup>7</sup>. As we said above, it would come out as a great surprise if non-distillable states were found to violate some Bell's inequality—and this is in itself a good reason to investigate this problem.

We want to conclude on Bell's inequalities. These inequalities, initially derived to quantify the counter-intuitive features of quantum correlations, are acquiring the status of 'witnesses for useful entanglement'. It had already been shown that the violation of a multi-qubit inequality is a sufficient condition for security of key-distribution protocols [31]. A link between Bell's inequality violation and communication complexity has also been noted [32]. Now we have proved that if a  $N$ -qubit state violates any of the inequalities found in [20, 21], then its entanglement can be distilled.

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<sup>7</sup> We note that there are examples of states violating the CHSH inequality although satisfying the reduction criterion. The violation of the latter is one of the most powerful calculable sufficient criteria for bipartite distillability.

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